Exercise 1 (Jacobi and SOR method - example) (10 points)

The discretization of the Poisson equation $-\Delta u = f$ over the unit square with simple finite elements (or finite differences) and a uniform grid gives a sparse $n^2 \times n^2$ matrix A that can be generated in Matlab using the command `gallery('poisson',n)`.

a) Examine the number of required iterations for the Jacobi method and the Gauss-Seidel method when solving $Au = f$ for the right-hand side

$$f = \frac{1}{(n+1)^2} \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}.$$

Use the methods of exercise `indir02_eng` to do this. As the termination condition, use a tolerance `tol` of 10^{-5} for the relative residuum (in the Euclidean norm). Use the zero vector as start vector of the iteration.

Generate a plot showing the number of iterations vs. the dimension n, for $4 \leq n \leq 18$ (set the maximum number of iterations to perform to 1000).

b) Solve the system $Bu = f$ with $B = A + 2I$ (I: identity matrix) under the same conditions as in a). How does the number of required iterations change? Explain the observed behavior.