Exercise 1 (Implementation cg method) **(10 points)**

cg algorithm:

- Initialize:

  Let u_0 be a start vector
  $r^0 = f - Au^0$
  $p^1 = r^0$
  $\rho^0 =< r^0, r^0 >$

- Iteration: $k = 1, 2, \ldots$

  $a^k = A \cdot p^k$
  $\alpha_k = \rho^{k-1} / < a^k, p^k >$
  $u^k = u^{k-1} + \alpha_k p^k$
  $r^k = r^{k-1} - \alpha_k a^k$
  $\rho^k =< r^k, r^k >$
  $p^{k+1} = r^k + \frac{\rho^k}{\rho^{k-1}} \cdot p^k$

Write a Matlab function with the function header

```
function [u,m] =
    solveCG(A, f, s, tol, max)
```

that solves the linear equation system $Au = f$ iteratively by using the cg algorithm (A must be a symmetric, positive definite $n \times n$ matrix). `tol` represents a given termination condition (see below), `maxit` is the maximum number of iterations to perform, s is the iteration start vector. The output arguments of the function are the number of iterations performed m and the approximate solution vector u.

$$\text{Termination condition: } \frac{\|f - Au\|_2}{\|f\|_2} \leq \texttt{tol}$$

Test your implemenation by solving $Ax = b$, A the Poisson matrix, $b = (1, \ldots, 1)^T$.