Exercise 1 (Preconditioned cg method) **(10 points)**

a) Test Matlab's `pcg` method using the Poisson matrix A, obtained with the command

`A = gallery('poisson', k)`

for $k = 1, \dots, 25$ and the right-hand side $(1, \dots, 1)^T$. Let `tol` = 1e-4, `maxit` = 200, and call the method in the form

`[x, flag, res, iter] = pcg(A, b, tol, maxit, W).`

Use the following preconditioners: $W_I = I$, $W_J = D$, $W_{SGS} = (D+L)D^{-1}(D+L^T)$, and $W_{SSOR} = \frac{1}{2-\omega}(\frac{1}{\omega}D + L)(\frac{1}{\omega}D)^{-1}(\frac{1}{\omega}D + L^T)$ with $\omega = 1.5$. Plot the number of required iterations w.r.t. the number of unknowns in semi-logarithmic scale using `semilogx`.

b) In each step, the residual vector $r' = W^{-1}r$ must be computed with respect to the preconditioned system. Matlab offers the option to pass the positive definite preconditioner W in splitted form, i.e. $W = W_1 W_2$. Explain why passing W in suitably decomposed form offers significant computational advantages. Verify this by calling `pcg` with the alternative syntax `pcg(A, b, tol , maxit, W1, W2)`, and compare the performance of `pcg` using `tic` and `toc` with the calling syntax of **a)**.