

- a) Implement the 3-stage Runge-Kutta method with the Butcher table

0	0	0	0
$\frac{1}{2}$	$\frac{1}{2}$	0	0
1	-1	2	0
	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{1}{6}$

in a m-file with the signature $[\mathbf{t}, \mathbf{y}] = \text{RK3}(\text{FUNC}, \mathbf{t0}, \mathbf{tN}, \mathbf{y0}, \mathbf{N}, \mathbf{var})$. The input argument `FUNC` contains the handle that defines the right hand side function f , $[\mathbf{t0}, \mathbf{tN}]$ determine the time interval, $\mathbf{y0}$ is the starting (column) vector and \mathbf{N} denotes the number of time steps. `var` should be passed to the file `FUNC` as third argument (i.e. `FUNC(t, y, var)`). The output arguments are the vector of time steps $\mathbf{t} = (t_i)_{i=0, \dots, N}$ and the corresponding approximate solutions $\mathbf{y} = (y_i)_{i=0, \dots, N}$.

- b) Download the files `RK4.m`, `rk_comp.m` and `TESTODE.m` from the web page and save it in the same folder as your files from a). State the initial value problem that is solved in `rk_comp`. Then, test your code of a) by running `rk_comp`, and determine the numerical order of convergence for each of the methods from the output. Do the results coincide with the theoretical expectations?